# New Mathematical approaches in Electrocardiography Imaging inverse problem

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## Context and objectives

### Major objectives

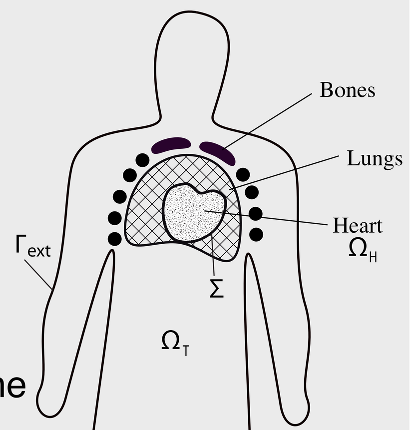
- ***Improve*** ECGI inverse problem reconstruction
- Introduce new mathematical approches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification apronches to deal with measurements errors

## Mathematical model

### Forward model

- If we know the heart potential we can compute the electrical potential

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla} u_\mathrm{T}) = 0, \text{ in } \Omega_\mathrm{T}, \\ \boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla} u_\mathrm{T}.\boldsymbol{n} = 0, \text{ on } \Gamma_\mathrm{ext}, \\ u_\mathrm{T} = u_\mathrm{e}, \text{ on } \Sigma. \end{cases}$$




### Inverse problem

- If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla} u_\mathrm{T}) = 0, \text{ in } \Omega_\mathrm{T}, \\ \boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla} u_\mathrm{T}.\boldsymbol{n} = 0, \text{ and } u_\mathrm{T} = T, \text{ on } \Gamma_\mathrm{ext}, \\ u_\mathrm{T} = ?, \text{ on } \Sigma. \end{cases}$$

## MFS approach

- Solve the linear system

$$\hat{A}\vec{a} = \vec{b}$$

$$\hat{A} = \begin{pmatrix} 1 & f(\|x_1 - y_1\|) & \cdots & f(\|x_1 - y_M\|) \\ \vdots & \vdots & \cdots & \vdots \\ 1 & f(\|x_N - y_1\|) & \vdots & f(\|x_N - y_M\|) \\ 0 & \frac{\partial f(\|x_1 - y_1\|)}{\partial n} & \cdots & \frac{\partial f(\|x_1 - y_M\|)}{\partial n} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \frac{\partial f(\|x_N - y_1\|)}{\partial n} & \cdots & \frac{\partial f(\|x_N - y_M\|)}{\partial n} \end{pmatrix} \quad \vec{a} = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_M \end{pmatrix} \quad \vec{b} = \begin{pmatrix} u_T(x_1) \\ \vdots \\ u_T(x_N) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$f(r) = \frac{1}{4\pi r}$$

$x_1, \ldots, x_N$ : Torso points

$y_1, \ldots, y_M$ : Heart points

- Regularization with CRESO

## Optimal control approach

- Poincaré–Steklov variational formulation of the inverse problem.
- Minimize the following energy functional

$$J(\lambda) = \frac{1}{2}\int_{\Omega_\mathrm{T}} (\boldsymbol{\nabla} u_\mathrm{D}(\lambda) - \boldsymbol{\nabla} u_\mathrm{N}(\lambda))^2.$$

Subject to

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla} u_\mathrm{D}(\lambda)) = 0, \text{ in } \Omega_\mathrm{T}, \\ u_\mathrm{D}(\lambda) = T, \text{ on } \Gamma_\mathrm{ext}, \\ u_\mathrm{D}(\lambda) = \lambda, \text{ on } \Sigma. \end{cases} \qquad \begin{cases} \operatorname{div}(\boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla} u_\mathrm{N}(\lambda)) = 0, \text{ in } \Omega_\mathrm{T}, \\ \boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla} u_\mathrm{N}(\lambda).\boldsymbol{n} = 0, \text{ on } \Gamma_\mathrm{ext}, \\ u_\mathrm{N}(\lambda) = \lambda, \text{ on } \Sigma. \end{cases}$$

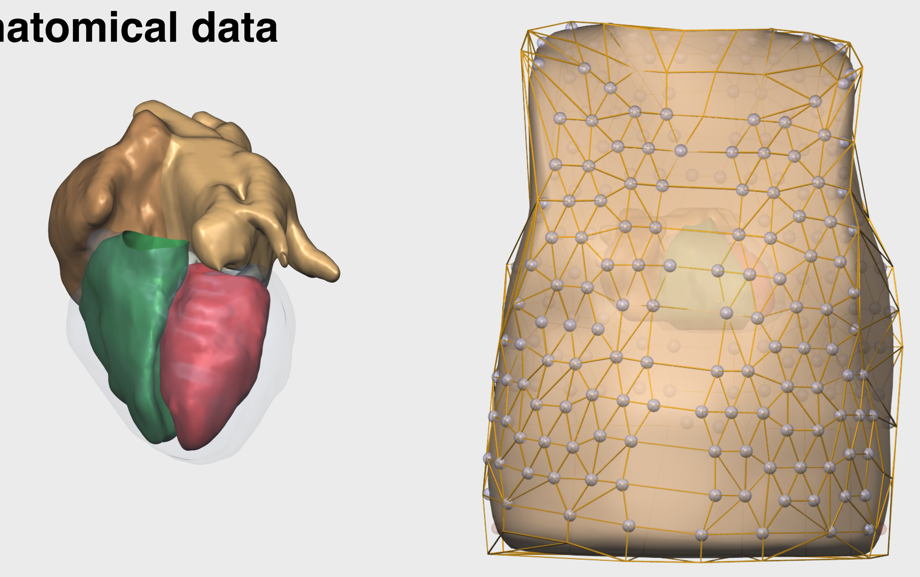
- Descent gradient methods

$$\boldsymbol{\nabla}_\lambda J(\lambda) = \boldsymbol{\sigma}_\mathrm{T}(\boldsymbol{\nabla} u_\mathrm{D}(\lambda) - \boldsymbol{\nabla} u_\mathrm{N}(\lambda)).\boldsymbol{n}_{/\Sigma}$$

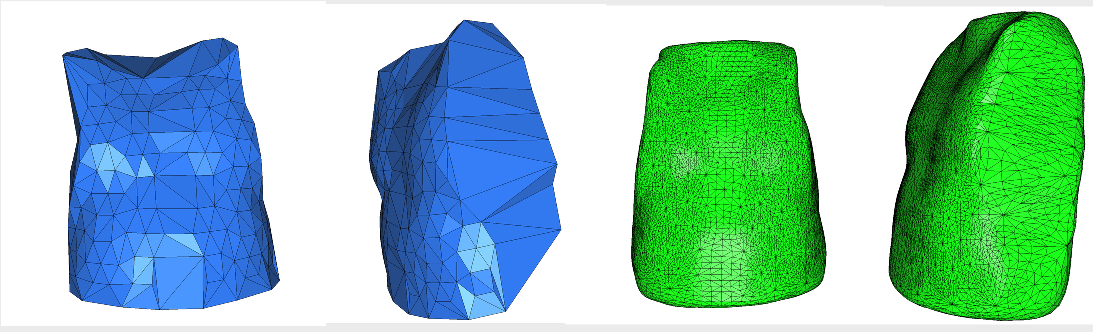
- Discretization with Finite elements method.

## In silico gold standard

### Anatomical data



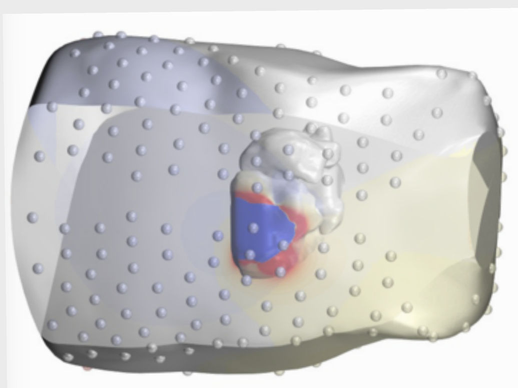
Computational heart and torso anatomical models + electrodes position

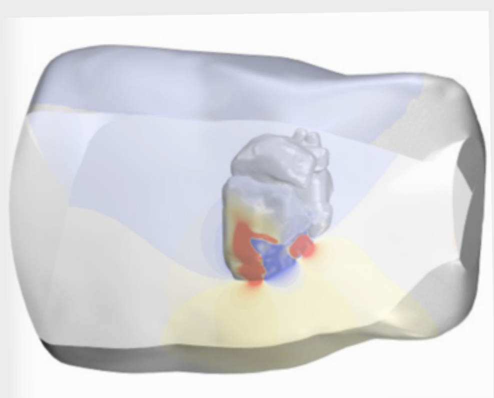


Computational torso meshes: 250 nodes mesh (blue). More accurate FE mesh with 6400 nodes (green)

### Simulated cases

- 6 single and double stimuli
- 14 reentry cases



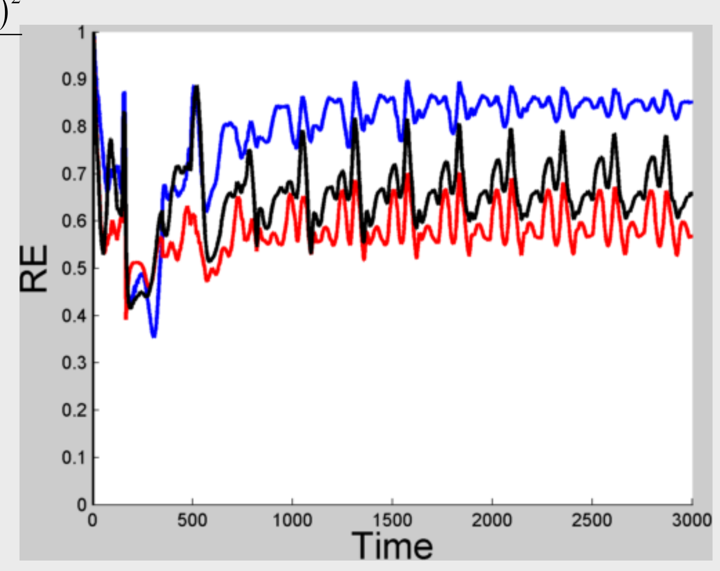


## Results

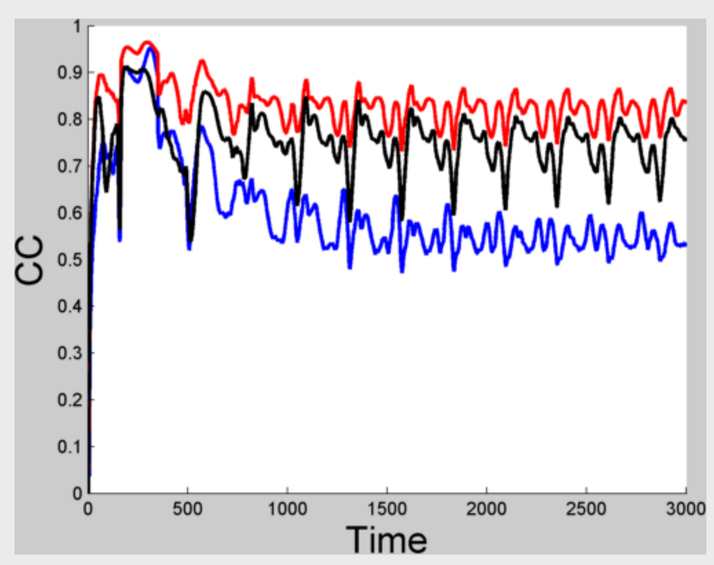
### Relative error and correlation coefficient

| Cases | metric | MFS + CRESO | O.C interpolated | O.C refined data |
|---|---|---|---|---|
| Single and double stimulus (6 cases) | RE | 0.81±0.04 | 0.71±0.02 | 0.59±0.06 |
| | CC | 0.57±0.07 | 0.7±0.03 | 0.8±0.04 |
| Re-entry (VT) (14 cases) | RE | 0.78±0.06 | 0.67±0.04 | 0.59±0.05 |
| | CC | 0.6±0.08 | 0.73±0.04 | 0.83±0.04 |
| All 20 cases | RE | 0.79±0.06 | 0.69±0.04 | 0.59±0.05 |
| | CC | 0.59±0.07 | 0.72±0.04 | 0.82±0.04 |

- $$RE = \sqrt{\frac{\sum_{i=1}^{L} (V_i^C - V_i^M)^2}{\sum_{i=1}^{L} (V_i^M)^2}}$$
- MFS + CRESO (blue line)
- Optimal Control + Tikhonov data from vest interpolated on Finer mesh (black line)
- Optimal Control + Tikhonov more rich data from Finer mesh (red line)

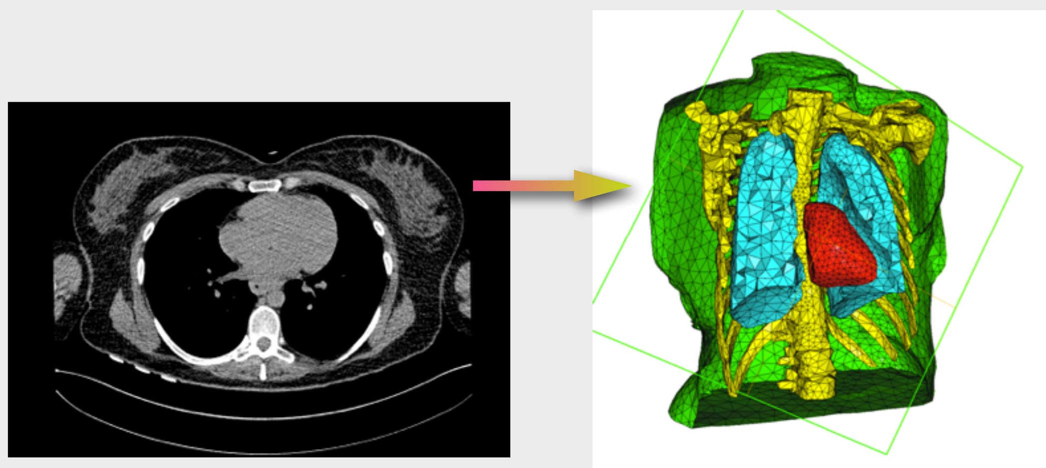



- $$CC = \frac{\sum_{i=1}^{L} (V_i^M - \bar{V}^M)(V_i^C - \bar{V}^C)}{\sqrt{\sum_{i=1}^{L} (V_i^M - \bar{V}^M)^2}\sqrt{\sum_{i=1}^{L} (V_i^C - \bar{V}^C)^2}}$$

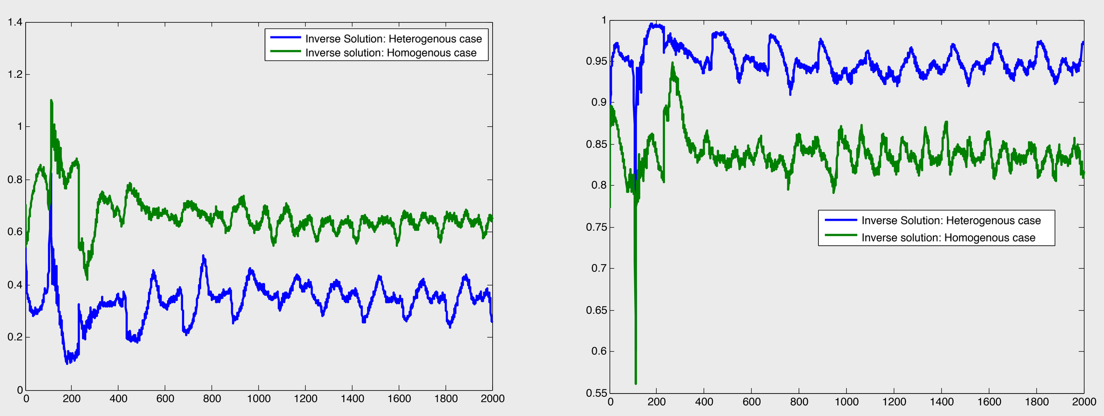



## Torso Heterogeneity effect

### Anatomical data

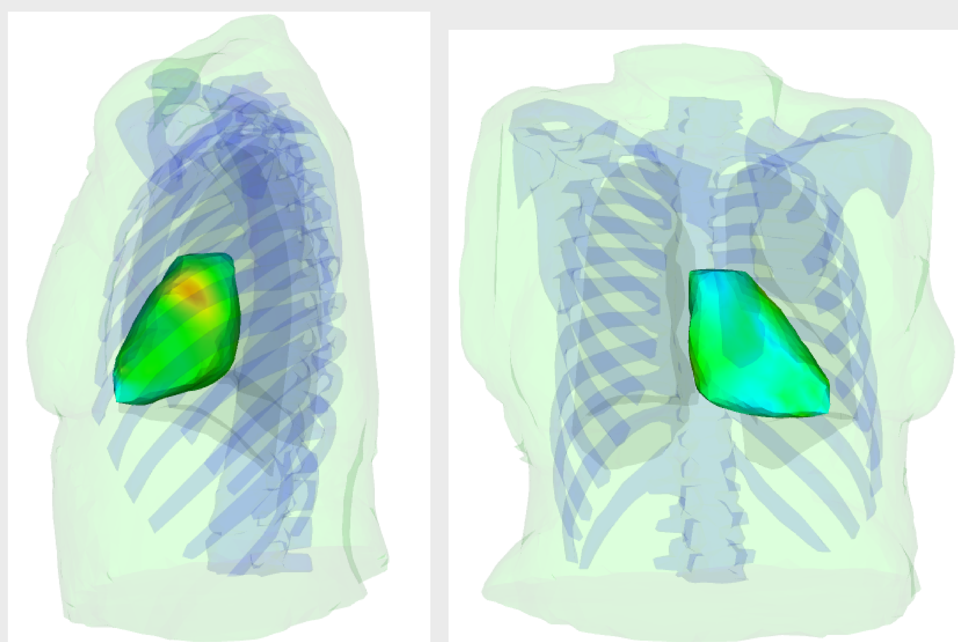


CT scan of a 43 years old women (left) and computational mesh obtained after segmentation (right )




Comparison of the optimal control solution for heterogeneous (bleu) and homogeneous (green) torso conductivities

### Space distribution of the error



Space distribution of the RE over time: Left (left) and right (right) ventricles views

### Remarks

- Introducing the torso heterogeneity is natural with FEM. also anisotropy could be introduced
- The error is more important in the left ventricle

## Conclusions

### Main results and perspectives

- New mathematical approches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79±0.06 to 0.59±0.05
  - CC was improved from 0.59±0.07 to 0.82±0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

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